MIMO Radar: A Statistical Approach to Radar

MIMO Radar Working Group:

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Outline

- Background and motivation
- Preview of concepts and results
- Signal model
- Information theory as a unified approach to radar analysis and synthesis
- Discussion of statistical MIMO
- Concluding remarks

A Bit of History: MIMO Communications

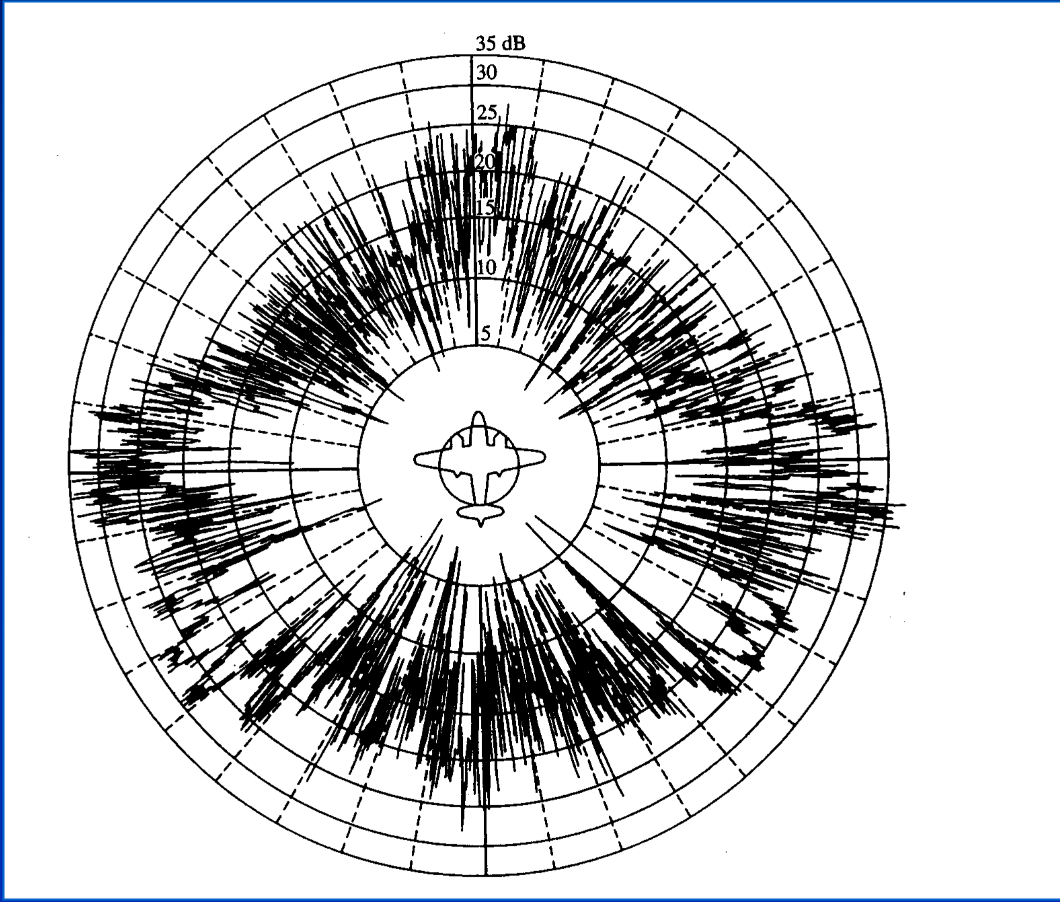
- In communications, the power-bandwidth exchange embodied in the expression for channel capacity is well-known and for a long time was considered an insurmountable barrier to high speed wireless communications.

- In 1996, a group of researchers from Bell Labs took the point of view (contrary to the accepted wisdom at the time) that the random multipath fading of the communications channel can be exploited by multiple transmit and receive antennas to yield many parallel physical channels through which data can be transmitted.

- In the few years, since the Bell Labs team has shown the feasibility of spectral efficiencies in the range of 20 bits/s/Hz (undreamed of until then), MIMO has become one of the most researched topics in communications. Currently, a variety of IEEE standards for high speed wireless are being modified to accommodate MIMO.

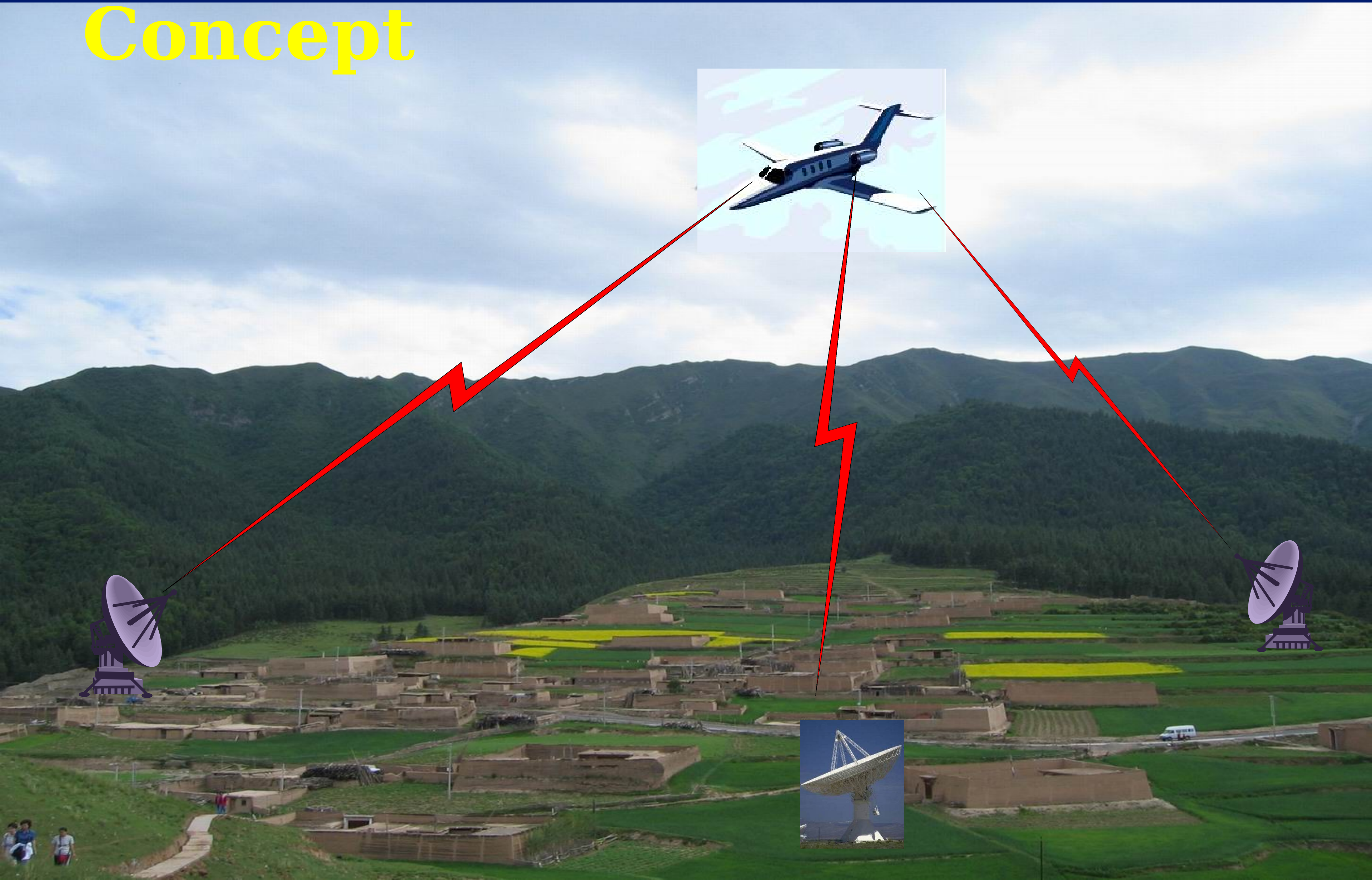
Motivation

- Radar targets provide a rich scattering environment.
- Conventional radars experience target fluctuations of 5-25 dB.
- Slow RCS fluctuations (Swerling I model) cause long fades in target RCS, degrading radar performance.
- Statistical MIMO radar is a new radar paradigm in which the angular spread of the target backscatter is exploited in a variety of ways to extend the radar's performance envelope.



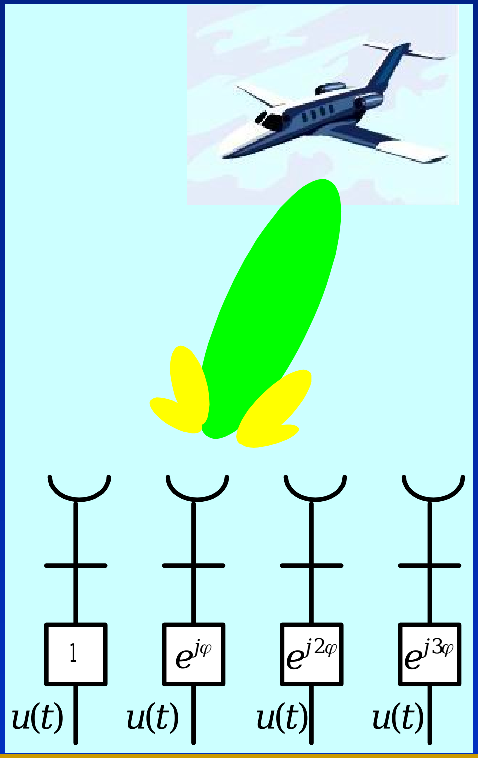
Backscatter as a function of azimuth angle, 10-cm wavelength [Skolnik 2003].

Statistical MIMO Radar Concept



Traditional Beamformer

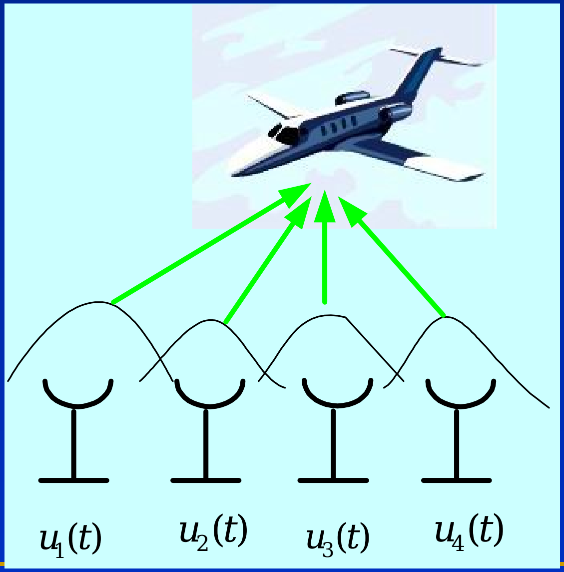
- The best known form of radar array is the beamformer. Time delays or phase shifts are used at the transmitter and at the receiver to focus a beam on the target and to cohere the signals across the receiver array.



Beamformer mode:
- Same waveform

MIMO Radar Terminology

- Recently, the radar community has been using the “MIMO” terminology to designate a radar that utilizes multiple transmitters to transmit independent waveforms. Main characteristics of MIMO radar:
  - Independent waveforms
  - Isotropic radiation – LPI advantage



MIMO mode:

- Independent waveforms

Recent Results in MIMO Radar

- Recent work has recognized the advantages of MIMO radar over traditional beamforming:
  - MIMO radar has been inspired by the French SIAR [Dorey et. al. 1989]
  - Optimization of transmitted waveforms [Guerci and Pillai 200?]
  - Performance bounds for MIMO/coherent mode [Fletcher and Robey 2003]
  - MIMO mode: SCR analysis [Rabideau and Parker 2003]

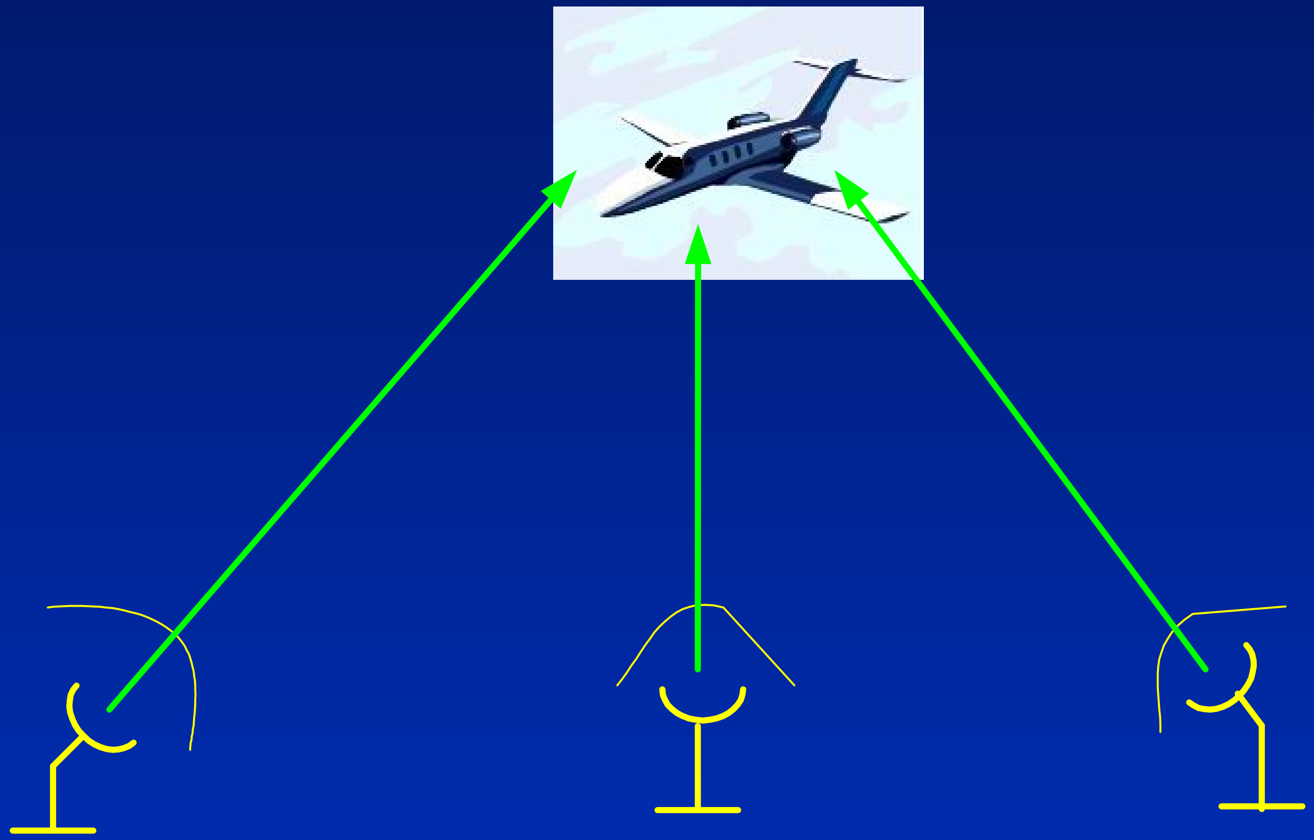
Statistical MIMO

- **Key Idea:** Reach the full promise of MIMO radar by capitalizing on the **target/channel angular spread** to improve radar performance.

  (Angular spread is a measure of the variability of the signals received across the array. A high angular spread implies low correlation between signals received at different array elements.)

- The MIMO concept is rooted in communication theory. In communication systems, MIMO capitalizes on scattering in the radio channel to achieve high-rate wireless communications.

Statistical MIMO



Statistical MIMO mode:

- Exploit angular spread

Statistical MIMO: First Steps

- Being a new form of radar, the full implications, significance and applications of statistical MIMO are largely unknown.

- Based on preliminary analysis, here, we have the modest goal of justifying gains in the major radar performance measures:
  - Resolution
  - Detection
  - Estimation

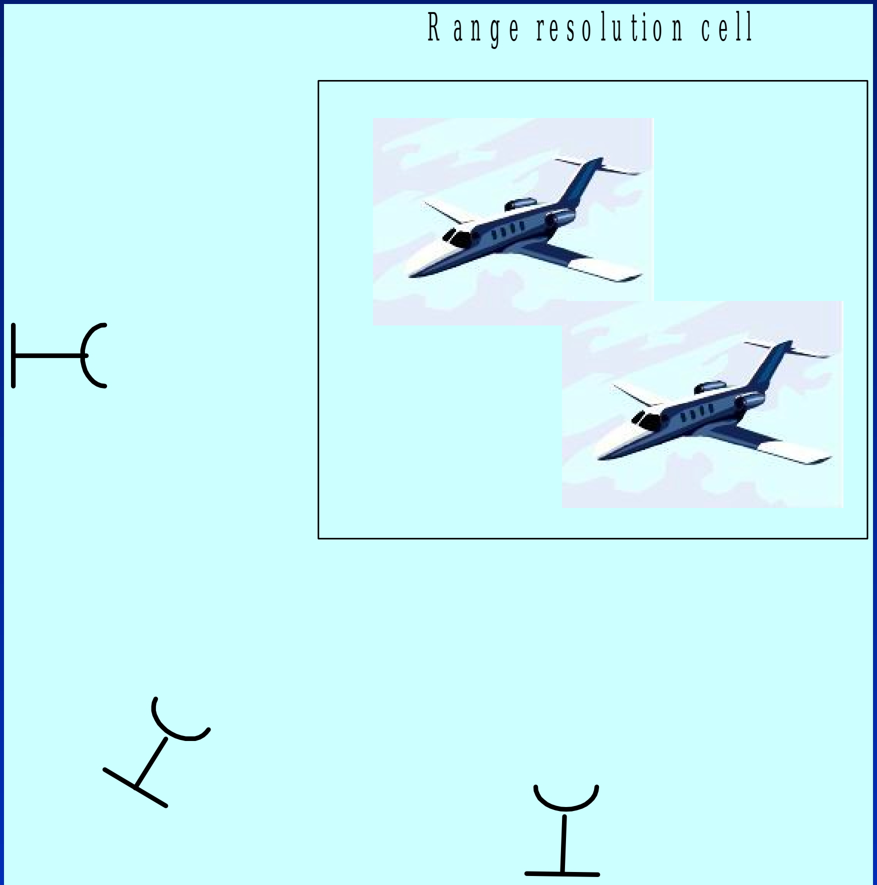
- Architecture naturally lends itself to independent radar apertures deployed over wide areas.

How is S-MIMO Different?

- Multichannel radar utilizing beamforming/MIMO mode is subject to performance degradation due to target fading.
- Current radar techniques employ a deterministic approach.
- In contrast, S-MIMO is capitalizing on target fluctuations (angular spread).
- Ground multipath, if available, is exploited to the radar’s advantage (rather than being regarded as an interference to be reckoned with).

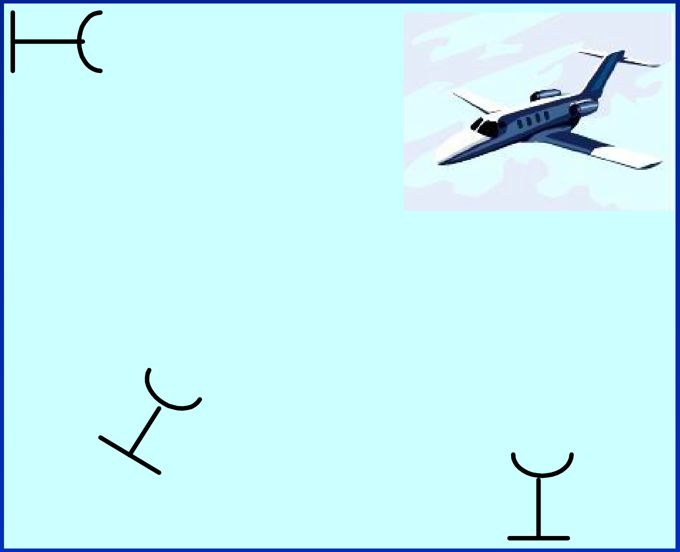
Resolution: Number of Targets

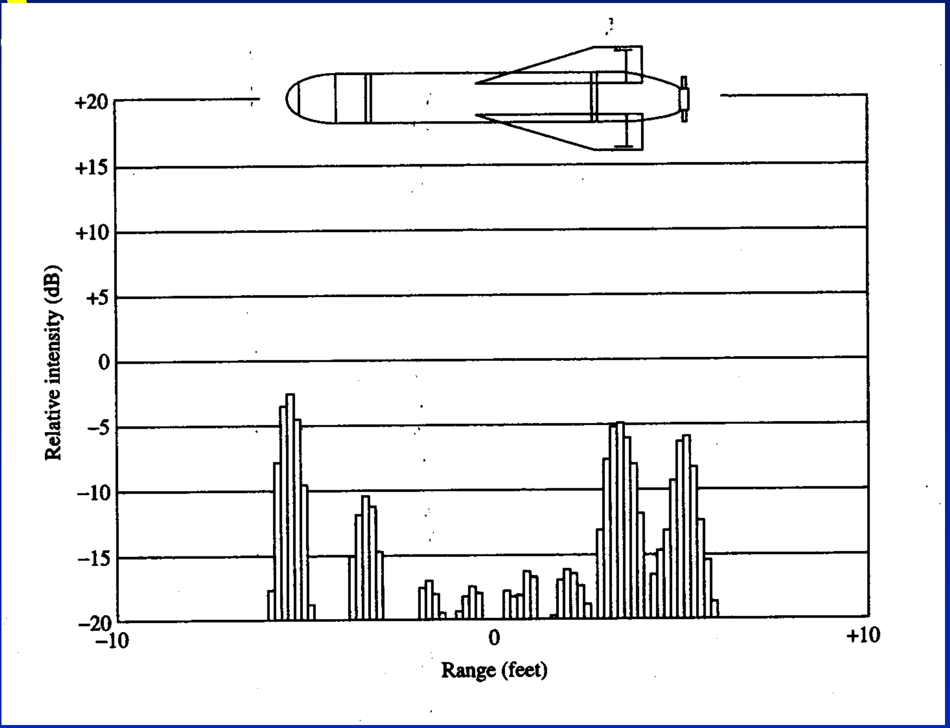
- Overcome range resolution bandwidth limitation by creating independent channels to targets in the same range resolution cell.
- Angular spread is created either by separation between elements or ground multipath.
- This has an effect of increasing the radar information similar to an increase in radar bandwidth



Resolution: Target Identification

- Independent channels to a target with multiple scatter centers supports target identification functions.





MIMO Resolution: How Does It Work

- MIMO achieves high resolution through the creation of independent signal channels for each target.
- Necessary conditions for that are:

(1) Angular spread of the cluster of targets created through antenna separation

$$\frac{D\Delta}{\lambda R} \geq 1,$$

D = target cross-range dimension

Δ = separation between array elements

R = distance to target, λ = carrier wavelength

or angular spread created by ground multipath.

(2) If Q = number of targets

M = numbr of transmit antennas

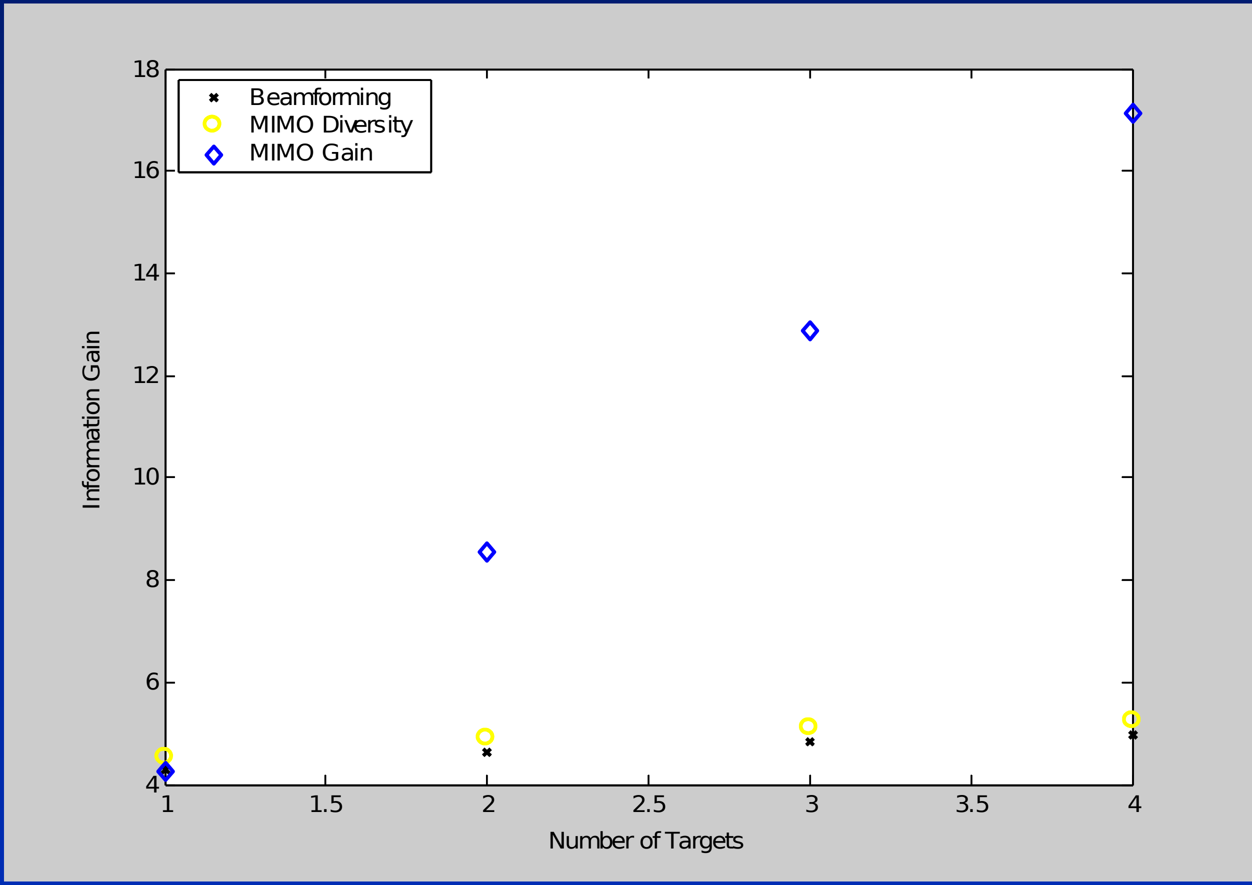
N = number of receive antennas

then the radar can resolve

$$Q \le \min(M, N)$$

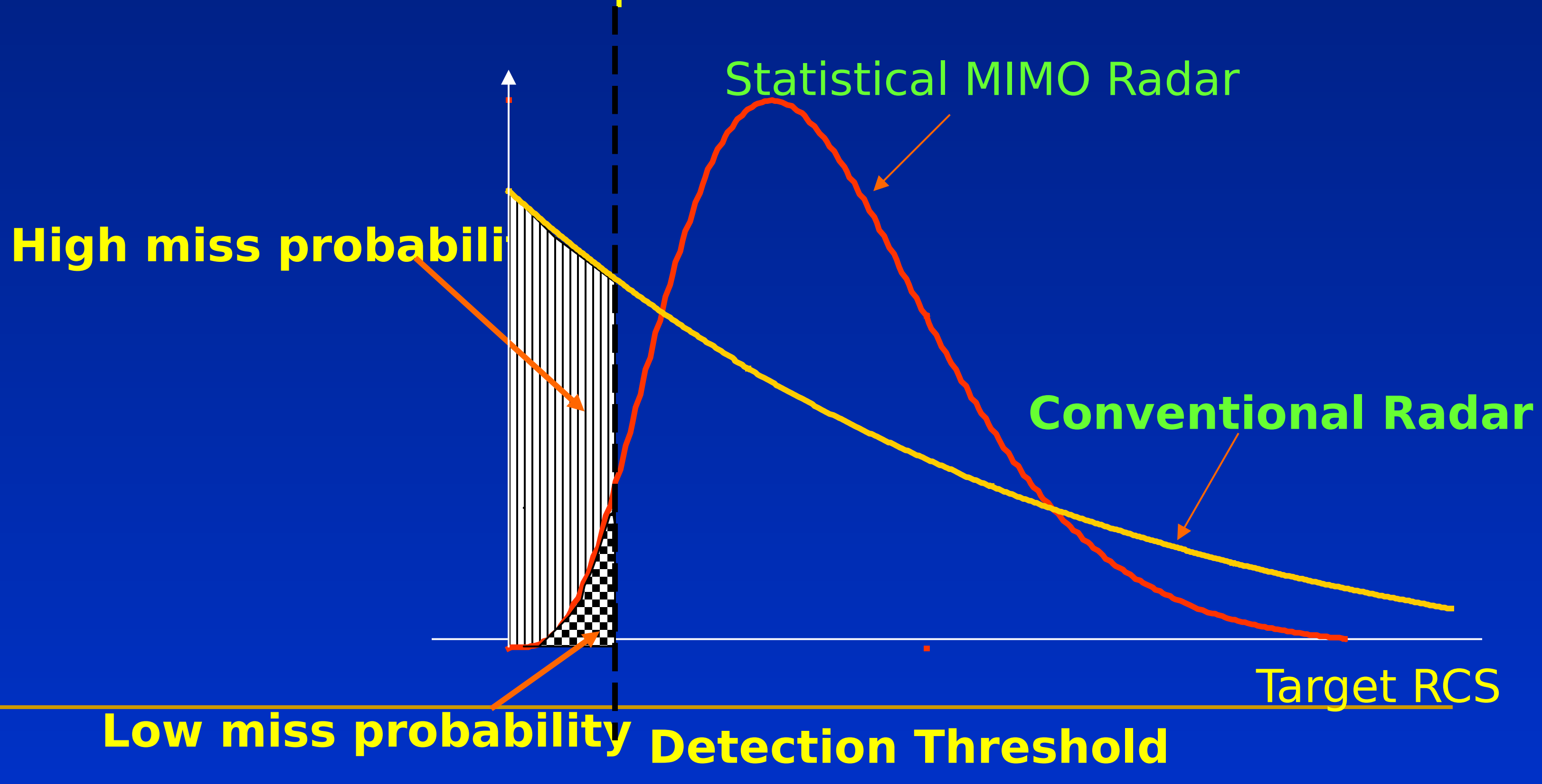
Resolution Gain

- Plot of "Information Gain," an indicator of the number of closely spaced targets.
- **MIMO Gain**: MIMO is used to resolve targets.
- **MIMO Diversity**: MIMO is used to enhance SNR.
- **Beamforming**: isotropic transmission (MIMO mode), beamforming on receive.



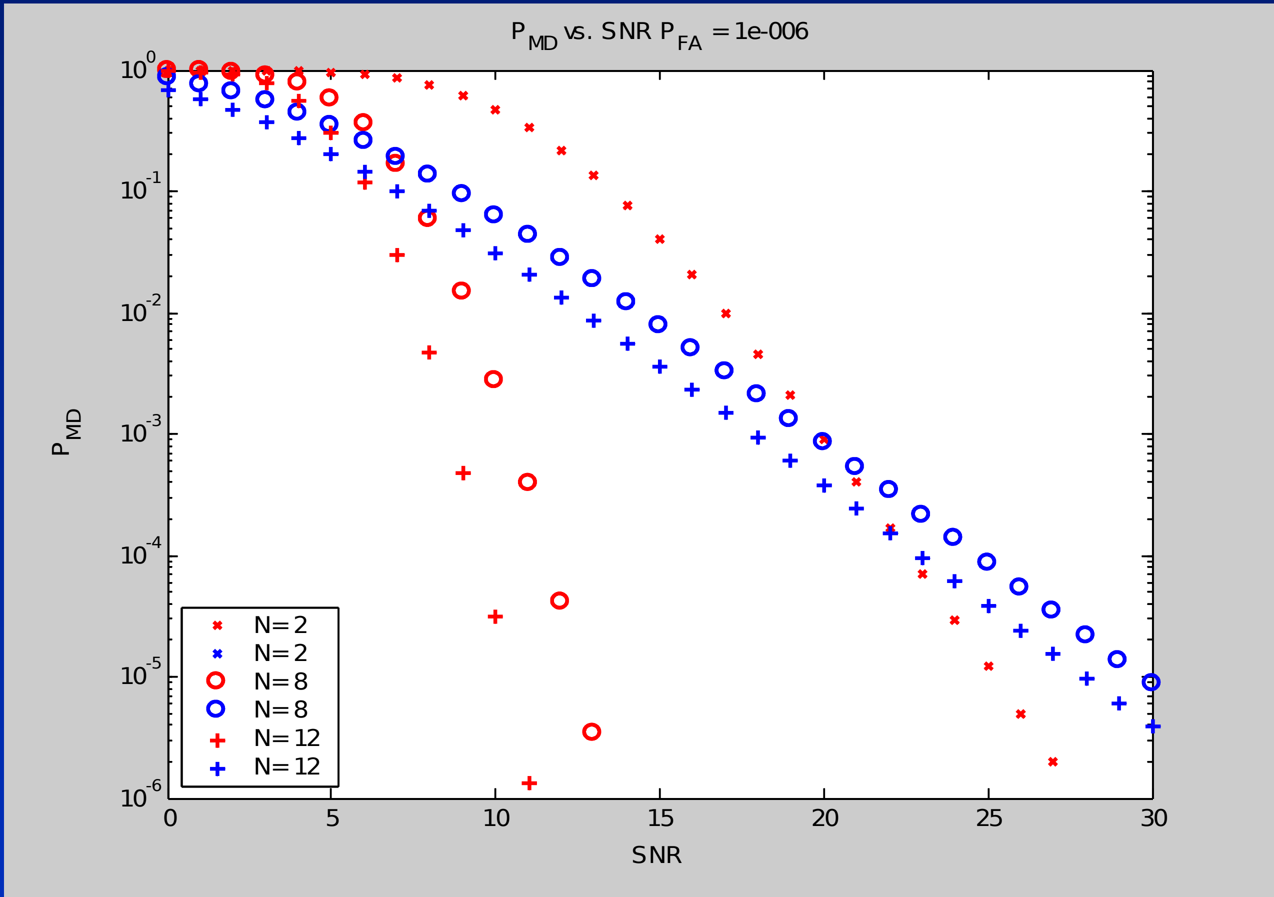
Detection and Estimation

- Statistical MIMO radar exploits target angular spread to reduce RCS fluctuations leading to improved detection and estimation performance.



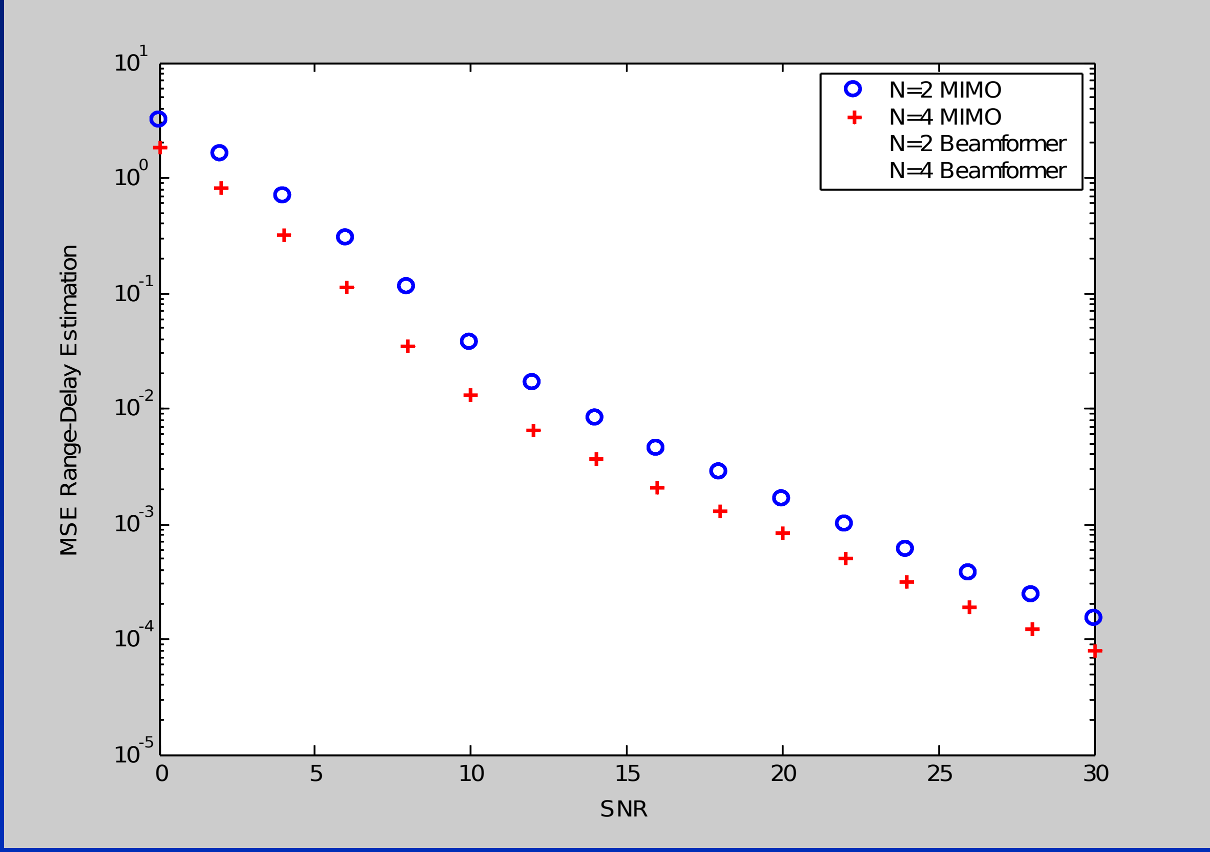
S-MIMO Detection Gain

- Plot of probability of missed detections, M=2.
- **MIMO Diversity** (red): MIMO is used to enhance SNR.
- **Beamforming** (blue): isotropic transmission (MIMO mode), beamforming on receive.



S-MIMO Range-Delay Estimation

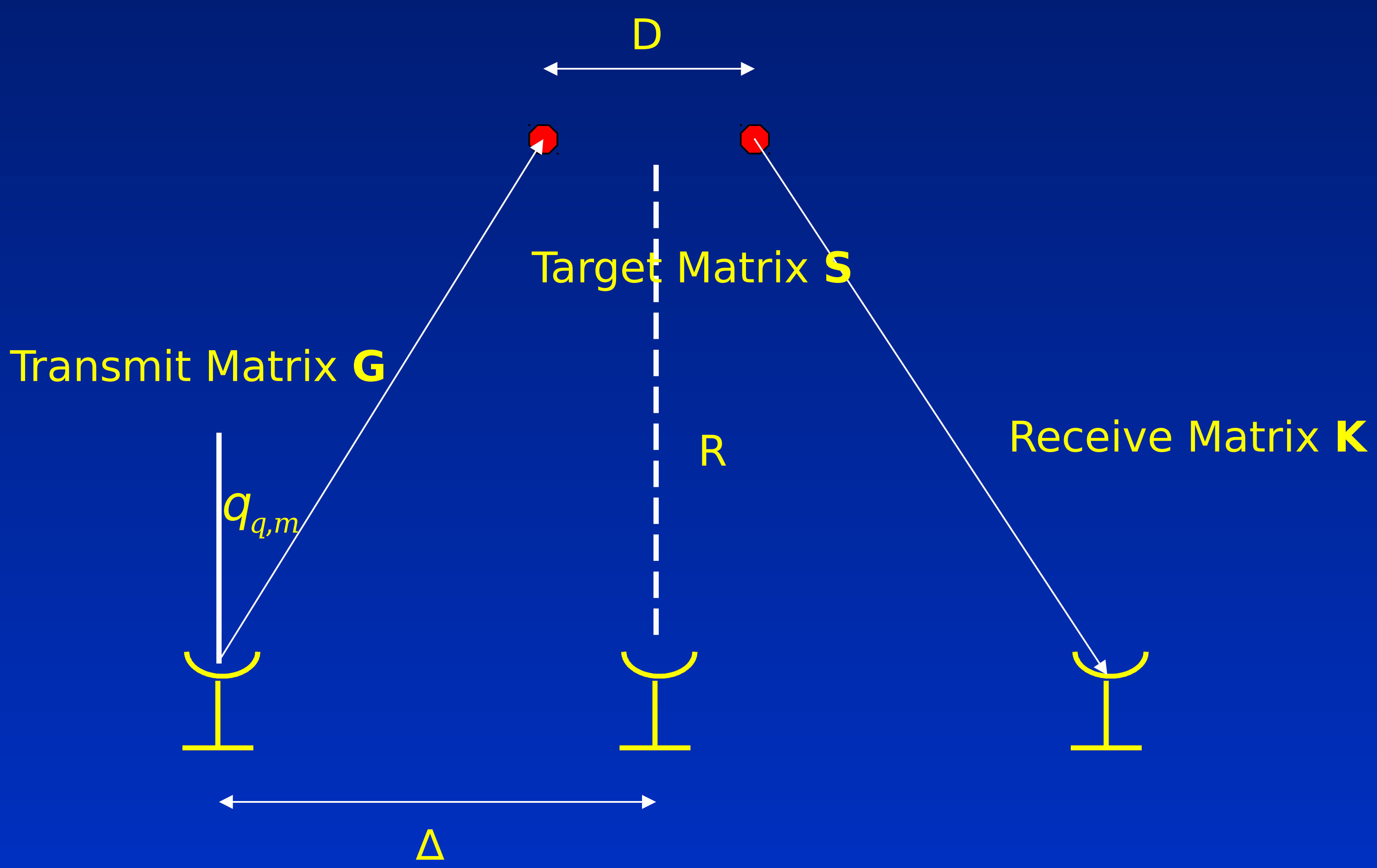
- Plot of estimation mean square error, M=2.
- **MIMO Diversity**: MIMO is used to enhance SNR.
- **Beamforming**: isotropic transmission (MIMO mode), beamforming on receive.



In the Reminder of the Presentation:

- Discuss the MIMO signal model
- Introduce a unified framework to the radar problem
- In this framework, explore relations between basic radar parameters such as power and bandwidth
- Further motivate the statistical MIMO radar concept
- Summarize

Signal Model



- What are the conditions on the target cross-range dimensions (or in the case of multiple targets, the cross-range distance between targets) and on the distance between radar elements such that the target is observed with a large angular spread?

Independent MIMO Channels

- It can be shown that a necessary condition for independence between MIMO channels to a target (or targets) is

$$\frac{D\Delta}{\lambda R} \geq 1,$$

where D is the target cross-range dimension, Δ is the separation between elements, R is the distance

- The MIMO channel model is then given by the $N \times M$ matrix

$$\boxed{\mathbf{H} = \mathbf{KSG} = \sum_{q=1}^{Q} a_q \mathbf{k}_q \mathbf{g}_q^T}$$

Q is the number of scatterers

MIMO Diversity Channels

$$\boxed{\mathbf{H} = \mathbf{K}\mathbf{S}\mathbf{G} = \sum_{q=1}^{Q} a_q \mathbf{k}_q \mathbf{g}_q^T}$$

Target RCS coefficients a_q are modeled as zero-mean, complex Gaussian i.i.d. random variables (Rayleigh fading target).

Put it All Together

- The received signal vector is given by the $N \times 1$ vector

$$\boxed{\mathbf{y} = \mathbf{H}\mathbf{u} + \mathbf{v}}$$

  where $\mathbf{u}$ is the vector of transmitted signals

- The rank of the matrix $\mathbf{H}$ meets the condition:

$$\operatorname{rank} \mathbf{H} \le \min\left(M, N, Q\right)$$

- The elements of the channel matrix $\mathbf{H}$ can be estimated by exploiting the orthogonality of the transmitted signals $\mathbf{u}$.

Radar Performance Measures

- Four performance measures capture the essence of radar operation [Siebert 1956]:

1. **Reliability of detection**: target competes with noise
2. **Accuracy** of target parameter estimation
3. **Resolution** of multiple targets
4. **Ambiguity** of estimates. The ambiguity problem has two aspects:
   - Noise ambiguity, where the noise is high and may interpreted as target returns
   - Ambiguity in the ability to measure a target parameter in the absence of noise.

Radar Signals Features

- Features of the radar signal that affect performance:

1. The signal **energy** (SNR) affects the reliability of detection, accuracy of the estimation, and to a lesser degree the resolution.
2. The signal **bandwidth** affects all performance measures
3. Signal **waveform** affects ambiguity and resolution
4. **Repetition** of waveform affects detection, estimation and ambiguity

Target Detection

- Simple signal model

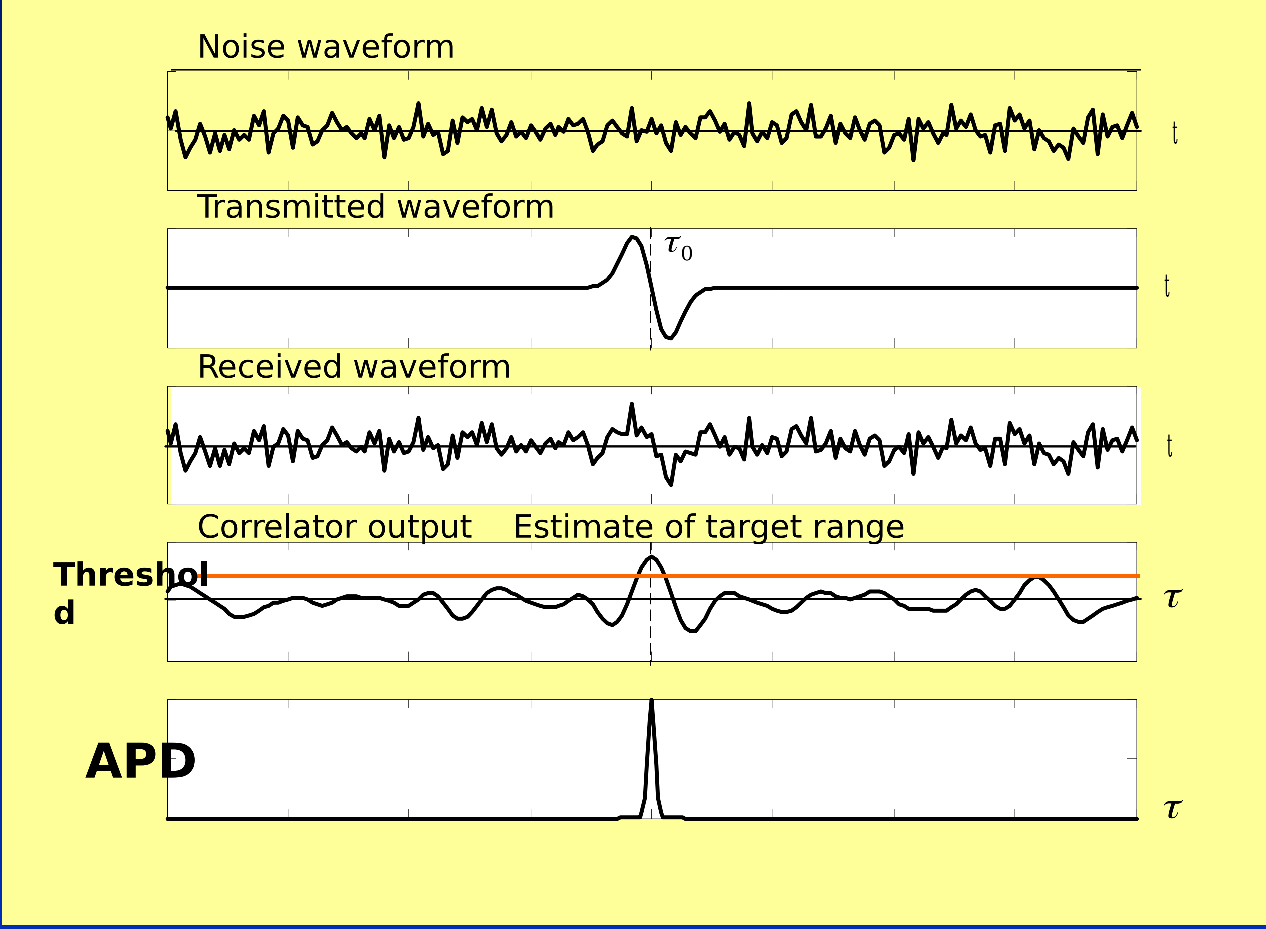
$$y(t) = u(t-\tau_0) + \text{noise}$$

- Correlation of the radar waveform with the received waveform over the range-delay interval Θ is sufficient for target detection

- A threshold can be set such that the probability of detection is maximized while the false alarms are kept at a desired level (Neyman-Pearson processor)

Range-Delay Estimation

- The range-delay for which the output of the correlator is maximized provides an estimate of the target range



A Posteriori Distribution

- The output of the correlator can yield the *a posteriori* distribution (APD) of the range-delay given the observation

$$p(\tau|y)$$

- The APD describes the probability that a target is at a specified range-delay. This is the most information that one can get from the observation. It takes into account any prior information on the location of the target.

- Let us consider the APD a little closer

What Can We Learn from the APD?

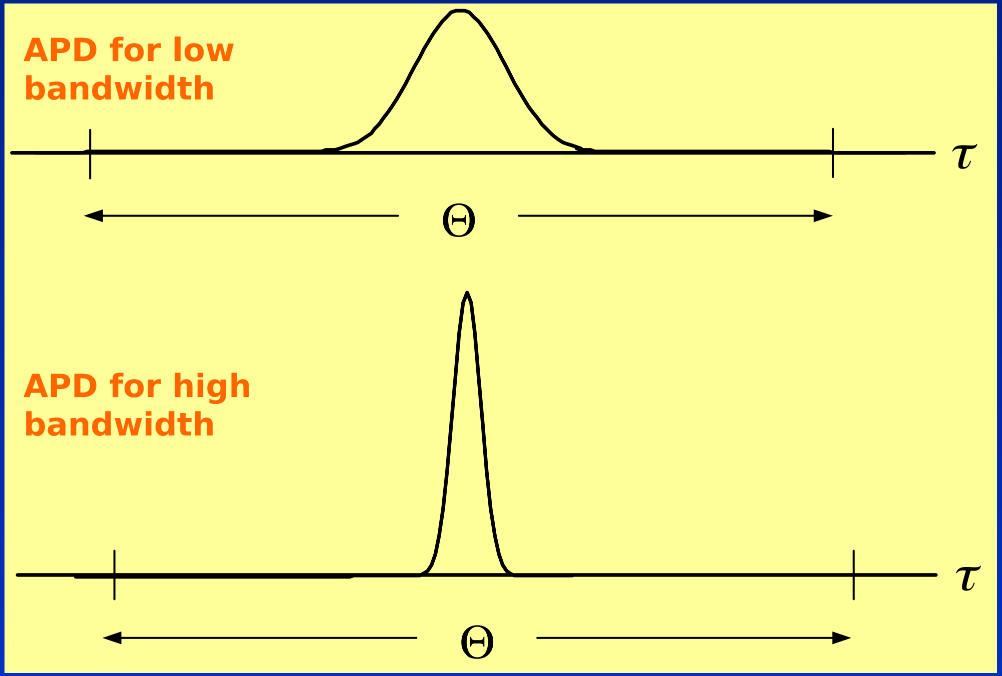
- Woodward (1952) has shown that the variance of the APD at the target's range-delay is given by

$$\sigma_\tau^2 = 1/(\beta^2\rho)$$

  where β is the radar waveform bandwidth and ρ is the SNR.
- This variance is a measure of the **accuracy** of the range-delay estimation.
- The expression reflects a **power-bandwidth exchange**, such that a deficit in each is compensated by increasing the other.

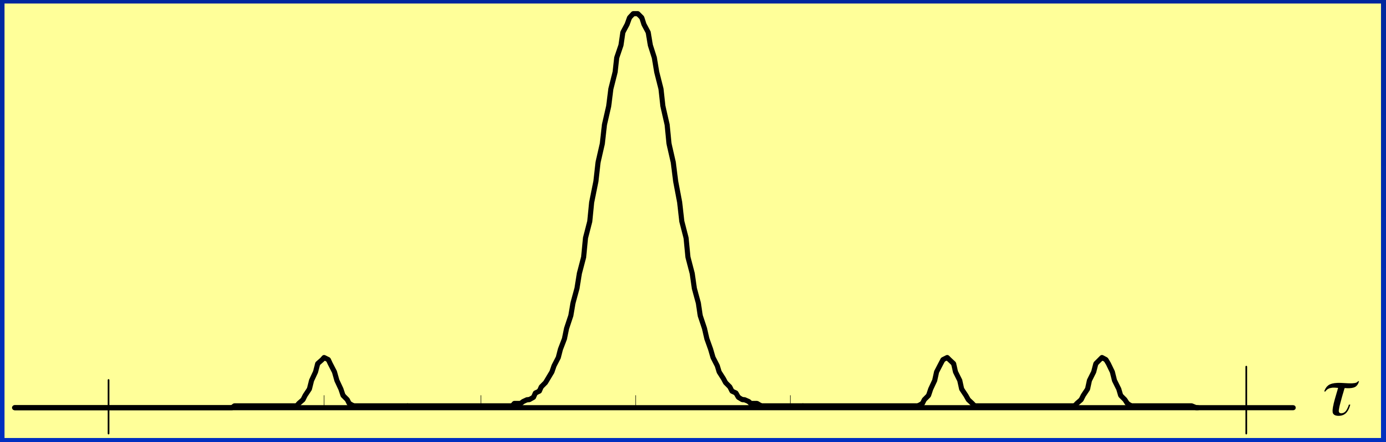
Relation to Resolution

- Relation to **resolution**: The product $\Theta\beta$ is approximately the number of APD's that can be resolved over the *a priori* range-delay interval Θ.



APD and Noise Ambiguity

- Relation to ambiguity: At low-SNR and high bandwidth β, there are higher chances that noise might "pop up."
- As the SNR increases, the noise peaks diminish in size compared to the target peak and the system transitions to a regime where true power-bandwidth exchange can take place.



A Unified Framework

- Traditionally, radar researchers have treated the detection and estimation problems separately (let alone resolution, noise ambiguity and waveform ambiguity).
- The APD provides a unified framework to detection and estimation.
- The APD also captures the **noise ambiguity** problem and the **resolution** factor $\Theta\beta$.
- Is there a representation of the radar information that unifies all of the above factors even better than the APD?

Radar Information Theory

- Radar IT is a field that has been only scarcely researched since Davies and Woodward's seminal work in 1952.
- It is of interest to us due to its ability to:
  - Unify seemingly disparate concepts
  - Provide insight into trade-offs among radar parameters
  - Guide us to optimal radar design
  - May help us draw from the rich experience gained with comm IT.
- In IT, entropy is a measure of the amount of uncertainty of a random variable x (similar to the thermodynamics concept). Given the pdf p(x), the entropy is defined

$$h(x) = -\int p(x) \log p(x)\, dx$$

Radar Information Gain

- The radar information gain is defined as the reduction in entropy (uncertainty) due to the processing of the radar observation.
- How is this concept useful? How does it relate to APD (or to detection and estimation)? How does it motivate MIMO radar?
- Consider the radar information gain for a single target

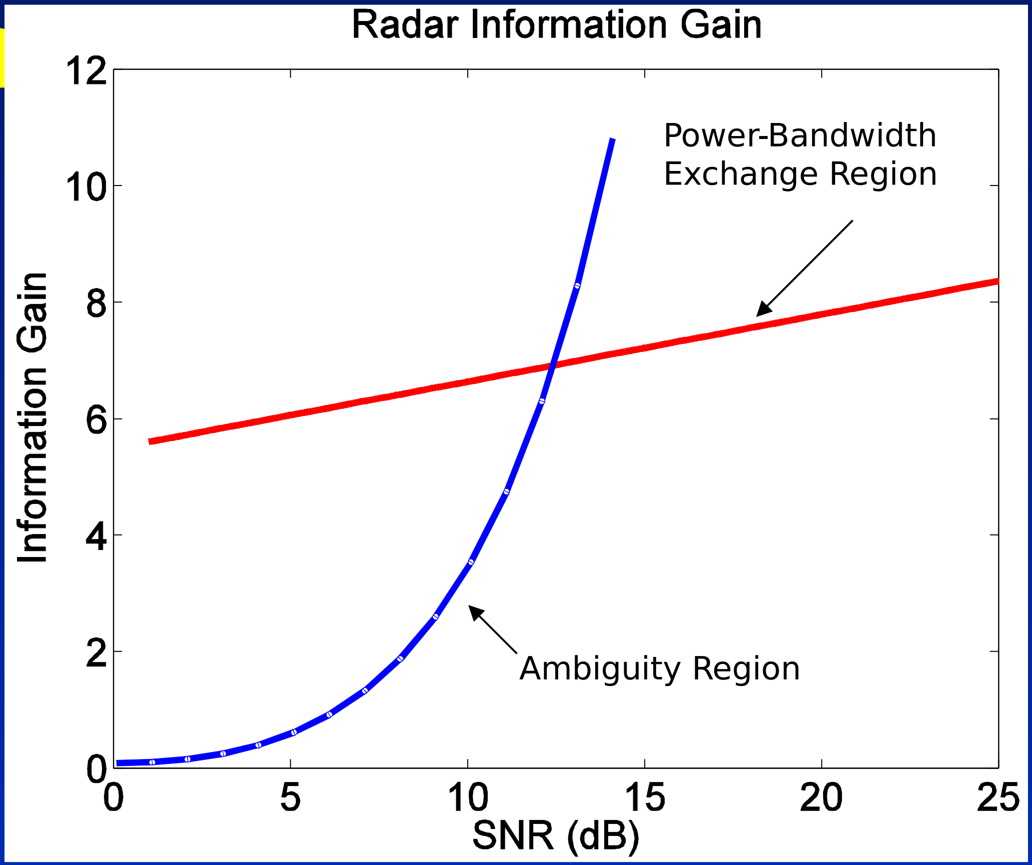
Resolution factor

Power-bandwidth exchange

$$I = \begin{cases} \frac{1}{2}\log\frac{Q^2 b^2 r}{2pe} & \text{high SNR} \\ \frac{1}{2}r - \frac{1}{2}\log\frac{2pr}{e} & \text{low SNR} \end{cases}$$

Noise ambiguity

Example of Information Gain



Does not capture waveform ambiguity

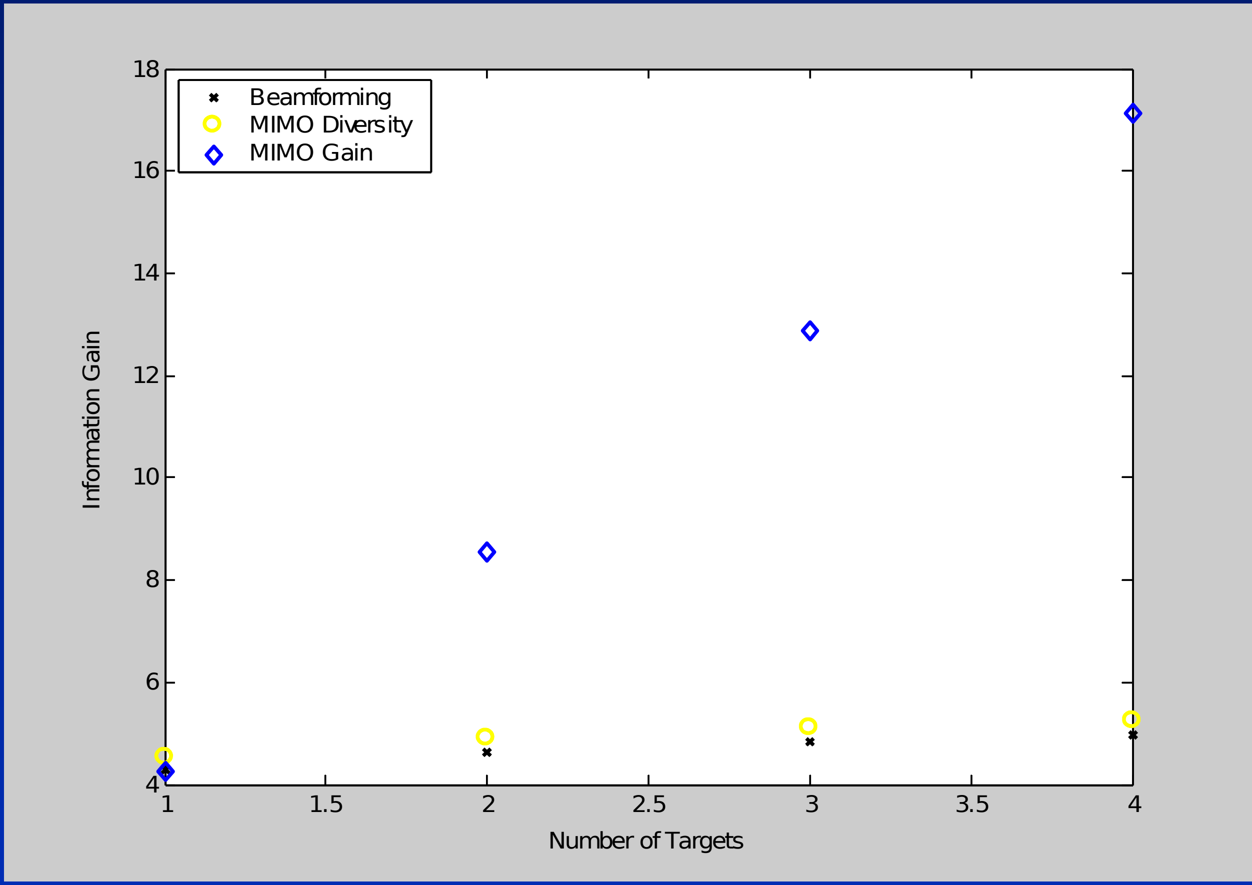
- The information gain increases linearly with the **actual** number of resolvable targets.

What We Learn from Radar IT?

- At sufficiently high SNR, the radar IG increases **logarithmically** with the SNR and the bandwidth (power-bandwidth exchange).
- The IG increases **linearly** with the **actual** number of resolvable targets.
- With conventional radar, the IG increases only **logarithmically** with the number of targets if the targets cannot be resolved in range.
- Based on preliminary analysis, with statistical MIMO, the IG increases **linearly** with the number of targets even when the targets are not resolvable in range, but are resolvable by the MIMO radar.

Resolution Gain

- Plot of "Information Gain," an indicator of the number of closely spaced targets.
- **MIMO Gain**: MIMO is used to resolve targets.
- **MIMO Diversity**: MIMO is used to enhance SNR.
- **Beamforming**: isotropic transmission (MIMO mode), beamforming on receive.



Discussion on Statistical MIMO Diversity

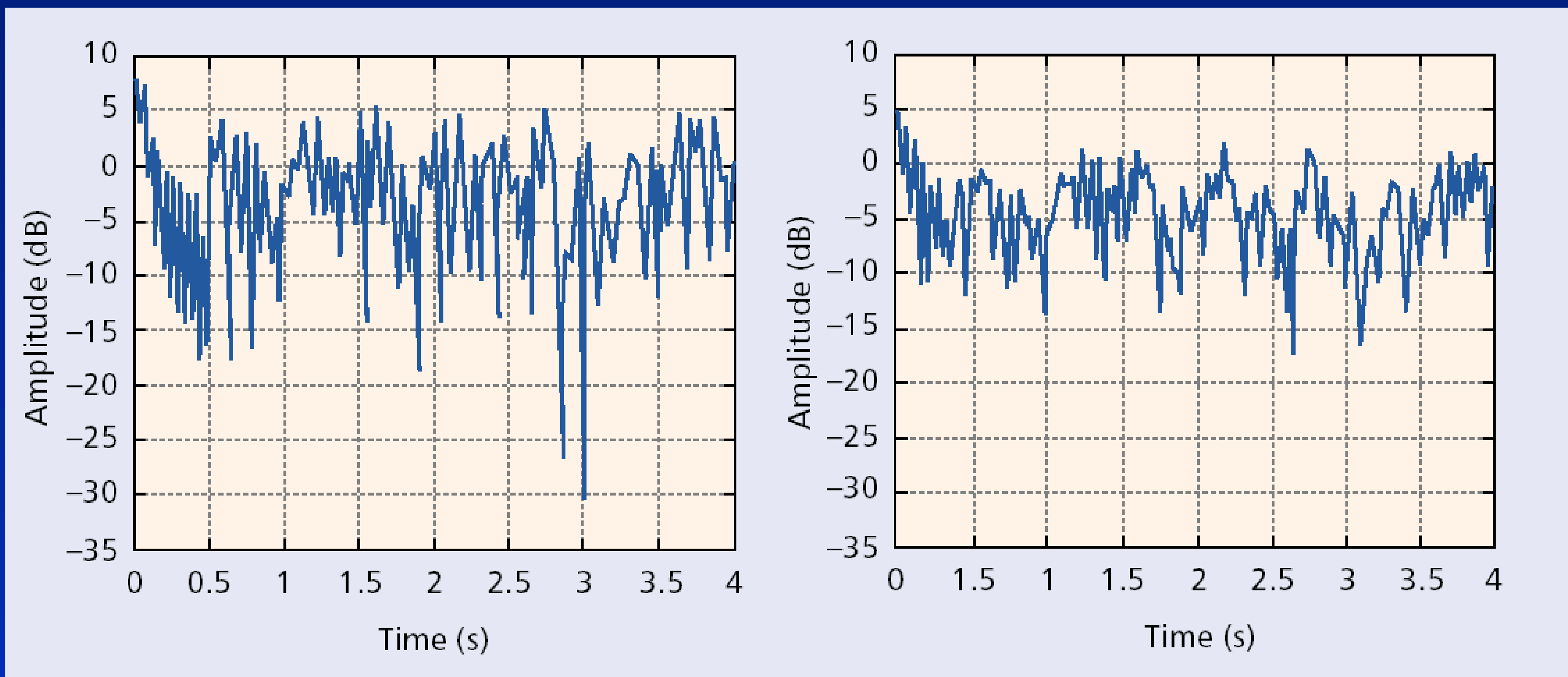
- It is well known that in conventional radar, in the absence of a diversity mechanism, target fluctuations degrade system performance (Swerling I, III models).

- It is also known that diversity mechanisms act to improve the radar performance. Common diversity mechanisms in radar:
  - Time diversity
  - Frequency diversity
  - Polarization diversity

- Statistical MIMO diversity overcomes problems that arise with other diversity techniques:
  - Time diversity takes up time (critical in search functions)
  - Frequency diversity requires wide bandwidth and it consumes a resource that can otherwise be exploited for range resolution
  - Polarization diversity is very limited.

Understanding Diversity

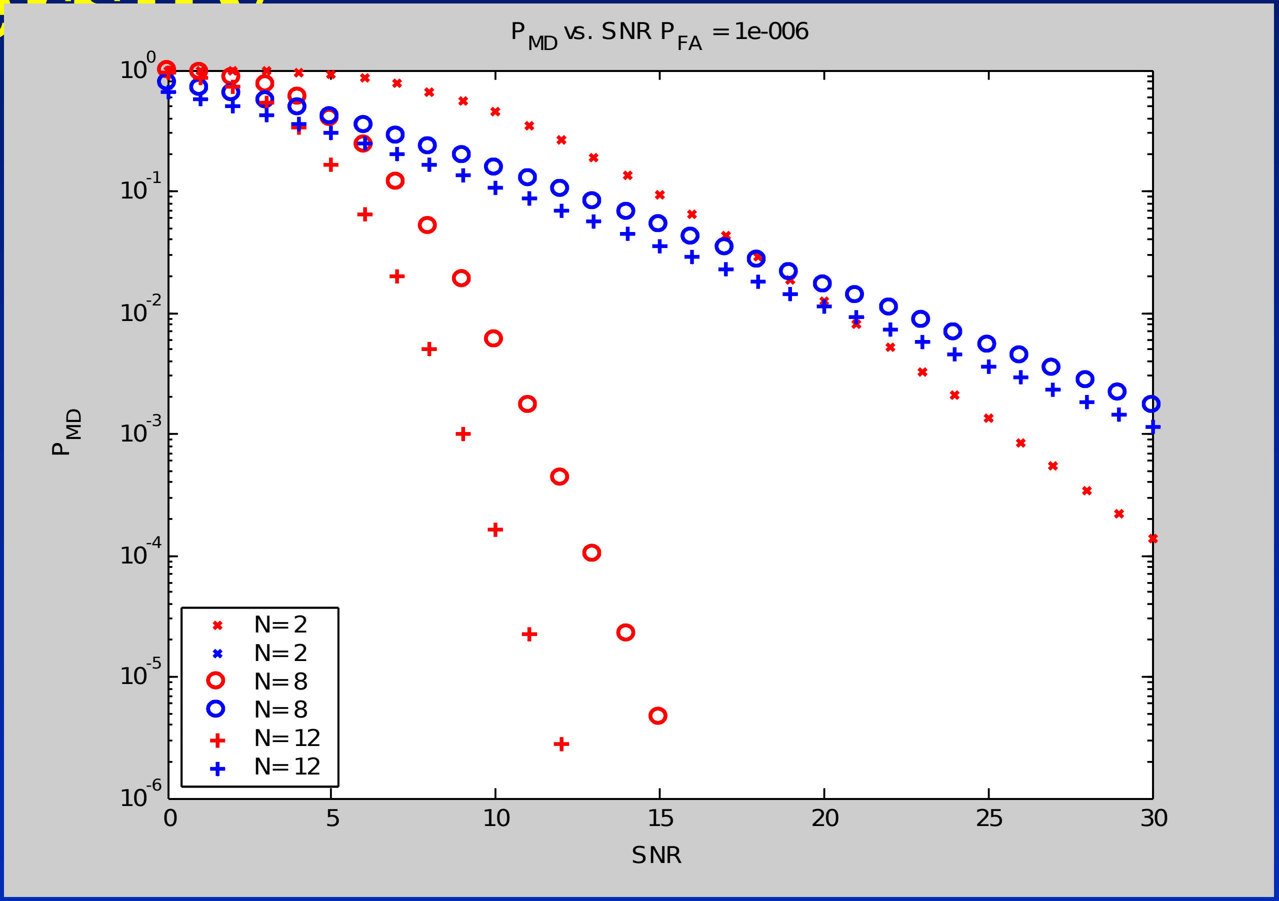
- Effect of diversity on signal envelope is to reduce the variance of the instantaneous SNR.



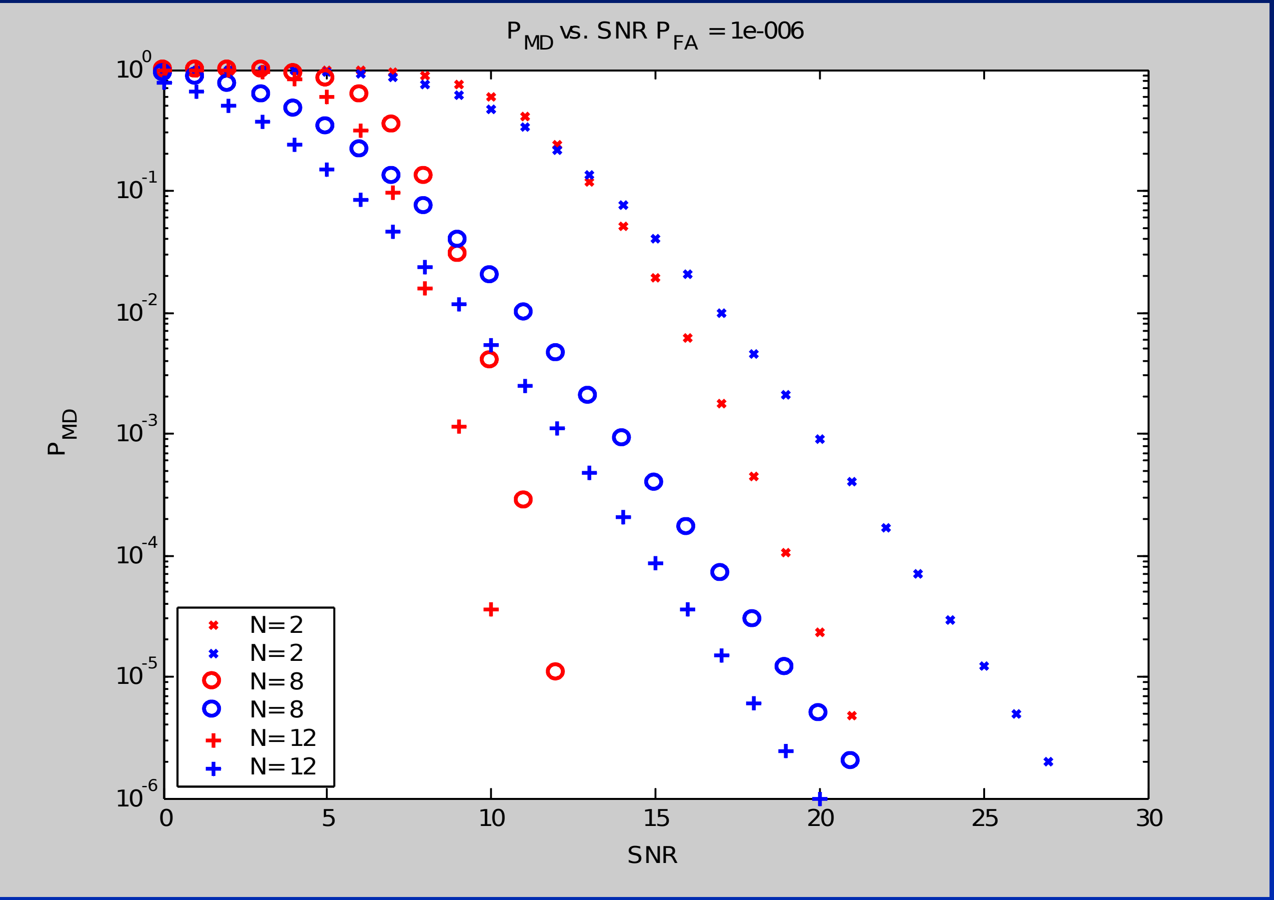
Single-path envelope

Two-path envelope

More Results on S-MIMO Diversity



Single transmit antenna



Four transmit antennas

Radar Modes with S-MIMO

- MIMO diversity mode for target detection and estimation
  - Isotropic radiation
  - LPI property
  - Coverage of large search volume
  - **Improved SNR**

- MIMO high resolution mode for target tracking and target identification
  - **Separation of scatterers within a range resolution cell.**

Concluding Remarks

- Statistical MIMO is a new concept for radar that may lead to new types of radars that capitalize on the statistical properties and angular spread of the radar targets and channel. Statistical MIMO may allow current MIMO radars to reach their full potential.

- Radar information theory provides a unified theory that may lead to advances in the analysis and synthesis of statistical MIMO and other types of radars.

- Preliminary results on statistical MIMO demonstrate an untapped potential to improve radar performance in terms of: detection reliability, estimation accuracy, reduce noise ambiguity, and increase resolution.

Statistical MIMO Working Group

- The full benefits, limitations and design issues of statistical MIMO are largely unknown at this time.

- Our working group was formed with the goal of exploring the untapped potential of statistical MIMO radar.

- The working group consists of the original inventors of MIMO communications from Bell Labs, Lucent Technologies and well known researchers in radar, signal processing and communications from University of Delaware, Lehigh University and New Jersey Inst. of Tech.